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Finite-size effects in multiquark droplets; An anatomical study

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Abstract

Strutinsky's averaging(SA) method is applied to multiquark droplets to systematically extract the smooth part of the exact quantal energy and thereby the shell correction energies. It is shown within the bag model that the semi-phenomenological density of states expression given upto curvature order is almost equivalent to the SA method. A comparative study of the bag model and the relativistic harmonic oscillator potential for quarks is done to investigate the quark mass dependence of the finite-size effects. It is found that there is an important difference between these two cases, which may be related to the presence/non-presence of the net spin-orbit effect.

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I. Introduction

The most interesting prediction in recent times is the Witten's conjecture[1] that the strange quark matter(SQM) might be the absolute ground-state of hadronic matter. Such a possibility has very significant astrophysical consequences[2]. In addition, probable formation and detection of small lumps of SQM in relativistic heavy-ion collisions is being promoted as an unambiguous signature of quark-gluon plasma[3]. In this context, stability of strangelets with baryon number $A \leq 100$ is of immense interest. In the determining the stability criteria of these small strangelets the finite-size effects such as surface and curvature play a significant role.

Farhi and Jaffe[4] were the first to consider the surface effects. Later on, Mardor and Svetitsky[5] suggested that curvature effects are also important. Recently, Madsen[6] has given a semi-phenomenological density of states(DOS) expression including terms upto curvature order for a system of quarks with mass m_q. He has shown within the bag model that this DOS expression reproduces quite well the exact shell model calculations irrespective of the quark mass m_q. (We hereafter refer to this model as quark liquid drop(QLD) model.) Further, an intriguing aspect is that the finite-size contributions in the QLD model do not seem to show a converging trend. More specifically, in the case of massless quarks the surface energy is zero, whereas the curvature contribution is significant. And in the case of massive quarks, the surface energy contribution is non-zero while the curvature energy contribution can be zero or positive or negative depending on the value of quark

mass m_q. (These features are clearly illustrated in Fig.1 of Ref.[6]). Consequently, the question of the importance of still higher-order terms such as Gauss curvature naturally arises. In other words, is the agreement between the QLD and shell model calculations accidental?

This question can be answered rather quantitatively with the help of the famous Strutinsky's averaging(SA) method[7]. In the context of nuclear physics, the SA method and the Wigner-Kirkwood(WK) expansion[8] have been shown[9] analytically to be equivalent. Therefore, by comparing the QLD model and the SA method we can determine the importance/non-importance of higher-order terms in the semi-phenomenological DOS expression of Madsen . In addition, the QLD model suggests that the finite-size effects depend on the value of the quark mass. As this can have very significant consequences in the study of quark-hadron phase transition[5], it would be interesting to know whether this finding is of general nature or is model-dependent. In this work, we mainly focus upon these two aspects of multiquark droplets. For this purpose, we consider the two potentials; the infinite square well(ISqW), namely a spherical cavity and the relativistic harmonic oscillator(RHO). The beauty of these two potentials is that both mimic quite adequately the asymptotic freedom and confinement properties of QCD. Moreover, they are analytically solvable.

In section II, we briefly discuss the nature of the shell structure in both these potentials. The Strutinsky's averaging method appropriate for quark systems is presented in section III and the results obtained are discussed in section IV. Finally, we summarize our findings in section V.

II. Confining potential, quark mass and shell structure

In regard to the shell structure in the ISqW and RHO potentials, at the outset the following observations can be made.

- In the case of the spherical cavity, the eigenvalues obtained exhibit splitting of levels such as $(p_{3/2}, p_{1/2})$, $(d_{5/2}, d_{3/2}) \cdots$ due to the spin-orbit coupling. On the otherhand, in the harmonic oscillator the eigenvalues are $j-$independent and hence no net spin-orbit effect on the energy spectrum. Thus, as expected, the shell structure depends on the nature of the confining potential.

- The other interesting observation is that the energy gap Δ_j between the $l = j + 1/2$ and $l = j - 1/2$ levels decreases as the value of quark mass m_q increases. This is illustrated in Fig.1 where we have plotted the eigenvalues obtained by the bag boundary condition[10] taking $m_q =$ 0, 150, 450 MeV. The zero of the scale is taken at the $0s_{1/2}$ level of massless case and therefore, what is plotted is $(\omega(\kappa) - 2.0428)$ in units of $\hbar c/R$. (While solving the boundary condition for massive quarks we have taken $R = 1$ fm). The decrease in Δ_j can be clearly noted in the $0p$ and $0d$ shells. In addition, due to the dependence of Δ_j on m_q, rearrangement of levels occur at higher j values as compared to the massless quark spectrum. The degree of rearrangement ofcourse depends upon the magnitude of m_q. In a recent work[11], it was found within the relativistic mean field theory of nuclei that the spin-orbit term is inversely proportional to the density dependent nucleon effective

mass. In analogy, we may expect Δ_j to decrease as the quark mass increases. Results displayed in Fig.1 seem to illustrate this effect. On the otherhand, in the case of RHO potential, the shell structure remains more or less the same expect for a relative shift in energy as m_q is varied.

Thus, already one can see that the nature of the confining potential and the quark mass have definite roles to play in determining the shell structure and thereby the stability of small multiquark droplets. In the following section, we shall outline the SA method and apply it to a system of quarks.

III. Strutinsky's averaging method for quark systems

The basic concept of the SA method[7] is that the total energy E of a given quantal system can be separated into a smooth part $\bar{E}$ and an oscillating part ΔE, i.e. $E = \bar{E} + \Delta E$. For a multiquark system consisting of more than one flavor, ΔE pertaining to each flavor needs to be calculated separately. For the sake of simplicity, we consider here a system of one kind of quarks with mass m_q confined either in a spherical cavity or in a harmonic oscillator potential. Then the exact single particle DOS is given by

$$g(\epsilon) = g \sum_i \delta(\epsilon - \epsilon_i), \tag{1}$$

where g is the degeneracy factor and ϵ_i are the eigenvalues of the potential under consideration. The total energy of the quark system is then given as,

$$E = g \sum_m \epsilon_m \quad ; \quad m \ = \ \text{occupied levels.} \tag{2}$$

Now, to evaluate the smooth part of the total energy E, Strutinsky proposed a numerical averaging of the single particle spectrum $g(\epsilon)$ by Gaussian smoothing funtions centered around ϵ_i and taken over a certain energy range γ. Thus, the smoothed level density is given as

$$\bar{g}(\epsilon) = \frac{g}{\gamma\sqrt{\pi}} \sum_{m=1}^{M} exp(-U_m^2) \sum_{i=1}^{p} C_i H_i(U_m), \tag{3}$$

where $U_m = (\epsilon - \epsilon_m)/\gamma$, M is the number of levels taken into consideration and $p = 6$[12]. The coefficients $C_i = (-1)^{i/2}/[2^i(i/2)!]$ for even i values, and $C_i = 0$ for odd i values. $H_i(U_m)$ are the Hermite polynomials. The average number of particles $\bar{N}$ and $\bar{E}$ are then defined as,

$$\begin{aligned} \bar{N} &= \int_{\infty}^{\bar{\lambda}} \bar{g}(\epsilon) d\epsilon, \\ \bar{E} &= \int_{\infty}^{\bar{\lambda}} \epsilon \bar{g}(\epsilon) d\epsilon. \end{aligned} \tag{4}$$

Having obtained the exact value E and the smooth part $\bar{E}$, we can determine the shell correction energies using the definition $\Delta E = E - \bar{E}$. In the following, we apply this method to the RHO and ISqW potentials.

A. Relativistic harmonic oscillator potential

Considering the quarks with current quark mass m_q to be confined by a scalar plus vector harmonic confinement of the type $\frac{1}{2}(1 + \gamma_0)(m_q + C_2 r^2)$, the corresponding Dirac equation can be solved analytially[13]. And the eigenvalues are determined by the equation

$$(\epsilon_N - m_q)(\epsilon_N^2 - m_q^2) = 4C_2(N + \frac{3}{2})^2, \tag{5}$$

with $N = 2n+l$. In the non-relativistic limit, $\epsilon_N = (N+3/2)\sqrt{(2C_2/m_q)}+m_q$ and in the limit of small mass, $\epsilon_N \simeq [2\sqrt{C_2}(N+3/2)]^{2/3} + m_q/3$. In analogy with the non-relativistic case, we take the potential strength parameter $C_2 = C_{20}A^{-2/3}$, where $A = N_q/3$ and N_q is the number of quarks. The quantity C_{20} can be related to the ground-state energy M_b of a $3-q$ system as $C_{20} = (1/9)(M_b/3)^3$. In the case of massless quarks, $M_b = 1085.5$ MeV and $C_{20} = (173.95)^3$ MeV^3; and for massive quarks, $M_b = 1672.45$ MeV and $C_{20} = (268.01)^3$ MeV^3.

The next important step is to choose an appropriate value of γ, and check that the smoothed part does not depend upon γ value. This can be done by using the so-called plateau condition[14]. In our study, we choose the $\gamma-$ parameter to be $\gamma = \gamma_0 C_{20}^{1/3} A^{-1/3}$. Then, the value of ΔE corresponding to the plateau region pertaining to each value of m_q and A is taken to be the 'physical' shell correction energy which then determines the value of $\bar{E}$. Further, it was found that the prominence of the plateau and its length depends upon the number of quarks N_q. As A decreases, it becomes somewhat tricky to fix the value of ΔE. The indeterminacy is however small.

It would be interesting to compare the so-determined values of $\bar{E}$ with those obtained using a Wigner-Kirkwood expansion and study the importance of higher-order finite-size effects such as Gauss curvature. The expressions for the quark number and the total energy using the WK expansion can be obtained analytically[15]. We state below the final expressions so-obtained upto $O(\hbar^2)$.

$$N_q = m_q^3\Big(\frac{2m_q}{C_2}\Big)^{3/2}(1+x)^{3/2}x^3 - \frac{3m_q}{4}\Big(\frac{2m_q}{C_2}\Big)^{1/2}(1+x)^{1/2}x,$$

$$E_{WK} = \frac{64}{1155}\, m_q^4 \Big(\frac{2m_q}{C_2}\Big)^{3/2} (1+x)^{1/2} \cdot \Big[\frac{945}{32}x^5 + \frac{2905}{64}x^4 + \frac{1135}{64}x^3 + \frac{3}{8}x^2 - \frac{1}{2}x + 1 - (1+x)^{-1/2}\Big] - \frac{2}{5}m_q^2\Big(\frac{2m_q}{C_2}\Big)^{1/2}(1+x)^{1/2}\ \Big[\frac{9}{4}x^2 + \frac{11}{8}x + 1 - (1+x)^{-1/2}\Big], \quad (6)$$

where $x = (\eta - m_q)/m_q$ and η is fixed by the number equation. Before, we present our results in this case, we would like to discuss the SA method in the case of ISqW potential.

B. Infinite square well potential

Considering N_q number of quarks with mass m_q to be confined in a spherical cavity of size R, the boundary condition[10] to be solved for determining the eigenvalues is

$$j_l(\omega_\mu) = -sgn(\kappa)\frac{\omega_\mu}{\omega+\mu} j_{\bar{l}}(\omega_\mu), \quad (7)$$

where $\omega^2 = \omega_\mu^2 + \mu^2$ with $\mu = m_q R$, and $\bar{l} = l - sgn(\kappa)$. Then, the total energy of the system using the bag model picture is,

$$E = \frac{\hbar c}{R}\sum_i \omega_i + \frac{4}{3}\pi R^3 B, \quad (8)$$

where B is the bag energy density. The equilibrium radius of the system is then determined by the saturation condition, $\partial E/\partial R\,|_{R_o} = 0$.

In this case, the single particle DOS $g(\omega) = \sum_i(\omega - \omega_i)$ is smoothed as in Eq.(3) and is given as,

$$\bar{g}(\omega) = \frac{g}{\gamma_\omega\sqrt{\pi}} \sum_{m=1}^{M} exp(-U_m^2) \sum_{i=1}^{p} C_i H_i(U_m), \quad (9)$$

with $U_m = (\omega - \omega_m)/\gamma_\omega$. Then the smoothed energy $\bar{E}$ can be obtained using the equation,

$$\bar{E} = \frac{\hbar c}{R}\bar{E}_\omega(R) + \frac{4}{3}\pi R^3 B, \tag{10}$$

where $\bar{E}_\omega = \int_\infty^{\lambda_\omega} \omega \bar{g}(\omega) d\omega$ and λ_ω is determined by the number equation. Here, the γ_ω is chosen as; $\gamma_\omega = \gamma_0 A^{-1/3}$.

With this parametrisation, we have checked the plateau condition in this case also for each value of m_q and A while calculating $\bar{E}$.

IV. Results and discussions

Having demonstrated a reliable way of extracting the smooth part from the exact quantal energy, we shall presently compare this SA method with the WK one in the case of RHO potential, and the QLD model in the case of ISqW potential.

In Table I, we have presented the energies obtained in the case of RHO potential for four values of baryon number A using the SA method [Eqs.(3-5)] and the WK method[Eq.(6)] and then are compared with exact ones[Eq.(2)]. The difference between $\bar{E}$ and E_{WK} is about 10 MeV for massless quarks and about 17 MeV for the massive ones. This may be attributed to the presence of higher-order $[O(\hbar^4)]$ correction terms. Notwithstanding this, we might say that the agreement is quite good. This then establishes the goodness of the WK expansion for further study of the quark mass dependence of surface and curvature energies.

In Table II, the results obtained in the case of the ISqW potential is given.

We have taken $m_q = 1$ MeV and $B^{1/4}$=145 MeV. One can immediately see that E_{QLD} agrees quite well with that of the SA method. Similar degree of agreement is also obtained for more massive quarks. This then demonstrates that the DOS expression upto curvature order of Madsen is quite adequate.

Consequently, an important question arises; Does the finite-size effects greatly depend on the mass of the quark as suggested by the QLD model? To answer this, we need to understand the following aspects of the liquid drop model(LDM) expansion of energy[16, 17]. The surface and curvature energy coefficients contributes respectively to orders of $A^{2/3}$ and $A^{1/3}$ in the LDM expansion of total energy:

$$E = a_v A + a_s A^{2/3} + a_{\rm cv} A^{1/3} + \cdots . \tag{11}$$

To these orders, there is a part coming purely from the surface and curvature terms of the DOS expression. But, there is also some extra contributions arising out of the finite size effect on the Fermi momentum k_F. (This can be noted from Eq.(4) of Ref.[17]). Because of this, the volume term in the DOS expression contributes towards $O(A^{2/3})$, $O(A^{1/3})$ $\cdots$, and similarly the surface term contributes towards $O(A^{1/3})$ and $O(A^0)$, and so on. Therefore, the vanishing of curvature term in the DOS expression does not necessarily mean that curvature energy coefficient is zero, but ofcourse its value is expected to be small. In view of this, we would like to extract the effective surface and curvature coefficients by making a least-squares fit to the total energies E_{QLD} calulated using three values of m_q. To do so, we used the QLD model with $B^{1/4} = 145$ MeV and have obtained energies for 101 number of droplets with baryon number A such that $6 \leq A \leq 1000$. The results so-obtained for

the energy coefficients are given in Table III for $m_q = 0,\ 150$ & 450 MeV.

It can be seen that the volume a_v coefficients obtained here agree with the bulk limits shown in Fig.1 of Ref.[6] for all the three values of m_q. Further, in the case of massless quarks the surface energy coefficient a_s is indeed zero, and $a_{\rm cv}$ is about 220 MeV. As the quark mass is increased to 150 MeV, the curvature coefficient has decreased as compared to the massless case illustrating the fact that the curvature term in the DOS expression is nearly zero for $m_q = 150$ MeV. Similarly, the value of a_s is now non-zero due to the presence of a surface term in the DOS expression. With further increase in m_q, the value of a_s increases, while $a_{\rm cv}$ remains more or less the same. Hence, although the curvature contribution in the DOS expression becomes negative as m_q increases, the effective curvature energy remains positive. Further, except for the massless case there is a converging trend in the LDM expansion of energy. It must be said here that the estimates obtained for higher-order coefficients such as Gauss curvature ($O(A^0)$) are not reliable as such terms were not included in the DOS expression. We have given them just to show that the three principal terms a_v, a_s and $a_{\rm cv}$ stabilises with respect to the number of parameters in the fit. Thus, there is a systematic dependence of surface and curvature energies on the mass of the quark. It must be stressed that the values obtained here are dependent on the bag constant value. Now, we are curious to see how far these findings are true in the case of RHO potential.

For this purpose, we repeated the same exercise taking the WK expansion of number and energy(Eq.(6)) for the same set of A and m_q. Here also we have checked the stability of the three energy coefficients with respect to the

number of parameters in the fit. The results so obtained are given in Table IV. It can be seen that the surface coefficient is independent of the mass of the quark. And the curvature coefficient increases from 150 MeV to 220 MeV as m_q varies from 0 to 150 MeV. With further increase in m_q, there is a slight decrease in $a_{\rm cv}$. In the case of massless quarks, it is somewhat disturbing to find the volume coefficient a_v less than 930 MeV. This can be rectified, as done in the case of bag models in choosing B, by appropriately choosing the potential parameter C_{20} so that the ud matter is unbound against nuclear matter. But, we feel our findings regarding the quark mass dependence of the finite-size effects shall remain unaffected as the surface and curvature terms are dependent only on the difference between the total energy and the bulk value.

Further, the surface energy coefficient remaining zero irrespective of the value of the quark mass needs more critical examination as it can have significant consequences for the phase transition studies. It may be recalled here that in the case of ISqW potential, the shell structure depends on the quark mass through the spin-orbit effect; whereas, in the case of RHO potential there is no such effect. Is this the underlying reason for the weak dependence of a_s and $a_{\rm cv}$ on m_q in the case of RHO potential? May be.

Thus, we have shown that the shell structure as well as the quark mass dependence of the finite-size effects are dependent on the nature of the confining potential.

VI. Summary

In summary, we have applied the Strutinsky's averaging(SA) method to multiquark droplets to systematically extract the smooth part of the exact quantal energy, and thereby the shell correction energies. It is shown in the case of bag model picture that the DOS expression given upto curvature order reproduces quite well the smoothed energies obtained by the SA method. Similarly, we found in the case of relativistic harmonic oscillator(RHO) potential, the Wigner-Kirkwood expansion with terms upto $O(\hbar^2)$ reproduces well the smoothed values of energy.

Having established the goodness of the asymptotic expansions in both the cases, we then made a comparative study of the two potentials, namely the spherical cavity and the relativistic harmonic potential, in regard to the quark mass dependence of the finite-size effects.

It was found in the case of the RHO potential that in contrast to the bag model picture, the surface and curvature coefficients are weakly dependent on the quark mass m_q. Further, the surface energy contribution is almost zero irrespective of the value of m_q. These differences may be traced back to the difference in the shell structure between these two potentials due to the presence/non-presence of net spin-orbit effect.

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TABLE CAPTIONS

Table I : Relativistic harmonic oscillator potential. Total energies calulated using the Wigner-Kirkwood(WK) expansion and the Strutinsky's averaging(SA) method are compared with the exact quantal ones for four values of baryon number A. The first set corresponds to the massless quarks, and the next one corresponds to quark mass $m_q = 150$ MeV.

Table II : Infinite square well potential. Total energies calulated using the quark liquid drop(QLD) and the Strutinsky's averaging(SA) method are compared with the exact quantal ones for four values of baryon number A taking the quark mass m_q=1 MeV.

Table III : Infinite square well potential. Values of the energy coefficients, volume a_v, surface a_s and curvature $a_{\rm cv}$, obtained by making a least-squares fit to the total energies calculated for 101 values of the baryon number A in the range 6-1000 using the quark liquid model. The three sets corresponds to quark mass $m_q = 0$, 150, & 450 MeV.

Table IV : Relativistic harmonic oscillator potential. Values of the energy coefficients, volume a_v, surface a_s and curvature $a_{\rm cv}$, obtained by making a least-squares fit to the total energies calculated for 101 values of the baryon number A in the range 6-1000 using the Wigner-Kirkwood expansion method.

The three sets corresponds to quark mass $m_q = 0$, 150, & 450 MeV.

FIGURE CAPTIONS

FIG.1 : Single particle levels obtained by solving the bag boundary condition are shown relative to the $0s_{1/2}$ level of the massless quark case in units of $\hbar c/R$. Index = 1, 2, & 3 corresponds to the quark mass m_q=0, 150 and 450 MeV respectively.

Table I

A	E_{ex}	E_{SA}	E_{WK}
6	5556.9	5466.2	5476.8
20	17597.3	17713.5	17723.6
40	34979.6	35126.0	35134.1
70	61013.4	61189.4	61196.4
6	9794.6	9653.1	9670.0
20	31275.6	31449.0	31460.7
40	62239.8	62456.6	62467.2
70	108629.3	108883.9	108899.7

Table II

A	E_{ex}	E_{SA}	E_{QLD}
6	6924.4	6948.9	6965.4
20	22580.2	22429.0	22437.7
40	44581.0	44406.5	44413.6
70	77231.6	77291.1	77296.4

Table III

# Param.	a_v	a_s	$a_{\rm cv}$	a_x	a_y
3	1090.3	1.258	218.0		
4	1090.4	−0.5139	225.3	−8.45	
5	1090.4	−0.0296	222.3	−0.838	−6.39
3	1212.4	265.0	91.8		
4	1211.8	273.7	55.9	41.4	
5	1211.9	272.8	61.4	27.7	11.5
3	1837.8	303.4	78.0		
4	1837.5	307.5	61.0	19.6	
5	1837.5	306.8	65.3	8.78	9.10

Table IV

# Param.	a_v	a_s	$a_{\rm cv}$	a_x	a_y
3	865.4	-2.68	161.1		
4	865.1	1.19	145.0	18.5	
5	865.2	0.015	152.4	-0.015	15.5
3	1542.9	-4.0	233.8		
4	1542.5	1.79	209.8	27.7	
5	1542.6	-0.096	221.7	-1.93	24.9
3	2175.7	-3.56	194.9		
4	2175.4	1.55	173.8	24.4	
5	2175.4	0.053	183.2	0.86	19.8

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